SHRI VISHWAKARMA SKIL UNIVERSITY

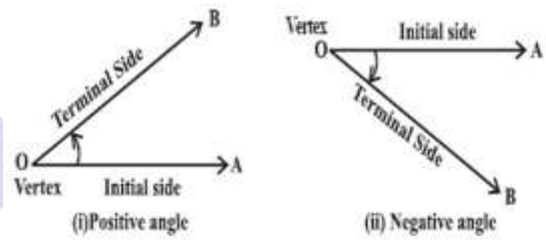
(CLASS NOTES)

SUBJECT: ZDSC-103(APPLIED MATHEMATICS-I)

FACULTY NAME: Dr. M.K. SRIVASTAV

UNIT-4: TRIGONOMETRIC FUNCTIONS

Angles: Angle is a measure of rotation of a ray about its initial point. The original ray is called the *initial side* and the final position of the ray after rotation is called the *terminal side* of the angle. The point of rotation is called the *vertex*. If the direction of rotation is anticlockwise, the angle is said to be positive and if the direction of rotation is clockwise, then the angle is *negative*.



Degree Measure: If a rotation from the initial side is $\left(\frac{1}{360}\right)^{th}$ of a revolution, the angle is said to have a measure of one degree, written as 1^0. A degree is divided into 60 minutes, and a minute is divided into 60 seconds. $\left(\frac{1}{60}\right)^{th}$ of a degree is called a *minute*, written as $1'$, and one sixtieth of a minute is called a *second*, written as $1''$.

Thus, $1^0 = 60', \ 1' = 60''$,

- $\text{Radian measure} = \frac{\pi}{180} \times \text{Degree measure}$
- $\text{Degree measure} = \frac{180}{\pi} \times \text{Radian measure}$

Radian Measure: Another unit for measurement of an angle is called the *radian* measure.

$$2\pi \text{ radian} = 360^0 \text{ or } \pi \text{ radian} = 180^0.$$

$$1 \text{ radian} = \frac{180^0}{\pi} = 57^0 16' \text{ appoximately.}$$

$$\text{Also, } 1^0 = \frac{\pi}{180} \text{ radian} = \frac{22/7}{180} \text{ radian} = 0.01746 \text{ radian appoximately.}$$

Example: The relation between degree measures and radian measure of some common angles are given in the following table:

Degree	30°	45°	60°	90°	180°	270°	360°
Radian	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

Example: Convert $40^0 20'$ into radian measure.

Solution: We know that $180^0 = \pi$ radian

Hence, $$40^0 20' = 40\frac{1}{3} \text{degree} = \frac{\pi}{180} \times \frac{121}{3} \text{radian} = \frac{121\pi}{540} \text{radian}$$

Example: Convert 6 radians into degree measure.

Solution: we know that π radian = 180^0

$$6\,\text{radians} = \frac{180}{\pi} \times 6 \text{ degree} = \frac{1080 \times 7}{22} \text{degree}$$

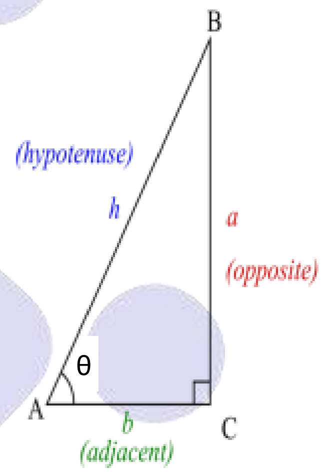
$$= 343\frac{7}{11}\text{degree} = 343^0 + \frac{7 \times 60}{11}\text{minute}$$

$$= 343^0 + 38' + \frac{2}{11}\text{minute}$$

$$= 343^0 + 38' + 10.9''$$

$$= 343^0 38' 11'' \text{ approximetly}$$

Topic 4.1. Trigonometry Ratio:



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\text{cosec}\theta = \frac{1}{\sin\theta} = \frac{\text{hypotenuse}}{\text{opposite}}, \sec\theta = \frac{1}{\cos\theta} = \frac{\text{hypotenuse}}{\text{adjacent}}, \cot\theta = \frac{1}{\tan\theta} = \frac{\text{adjacent}}{\text{opposite}}$$

- $\sin^2 x + \cos^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $1 + \cot^2 x = \text{cosec}^2 x$
- $\sin(-x) = -\sin x, \quad \cos(-x) = \cos x$
- $\tan(-x) = -\tan x, \quad \cot(-x) = -\cot x$
- $\sec(-x) = -\sec x, \quad \text{cosec}(-x) = -\text{cosecx}$

Review of ratio of some standard angles:

θ	0⁰	30⁰	45⁰	60⁰	90⁰
sin θ	0	½	1/√2	√3/2	1
cos θ	1	√3/2	1/√2	½	0
tan θ	0	1/√3	1	√3	∞
cot θ	∞	√3	1	1/√3	0
Sec θ	1	2/√3	√2	2	∞
cosec θ	∞	2	√2	2/√3	1

- $\sin(\frac{\pi}{2}-x)=\cos x$, $\cos(\frac{\pi}{2}-x)=\sin x$
- $\tan(\frac{\pi}{2}-x)=\cot x$, $\cot(\frac{\pi}{2}-x)=\tan x$
- $\sec(\frac{\pi}{2}-x)=\text{cosecx}$, $\text{cosec}(\frac{\pi}{2}-\text{x})=\text{secx}$
- $\sin(2\pi-x)=-\sin x$, $\cos(2\pi-x)=\cos x$
- $\tan(2\pi-x)=-\tan x$, $\cot(2\pi-x)=-\cot x$
- $\sec(2\pi-x)=\sec x$, $\text{cosec}(2\pi-x)=-\text{cosec}x$

The signs of other trigonometric functions in different quadrants are as follows:

	I	II	III	IV
sin x	+	+	–	–
cos x	+	–	–	+
tan x	+	–	+	–
cosec x	+	+	–	–
sec x	+	–	–	+
cot x	+	–	+	–

.

Topic 4.2. Compound Angles:

- $\sin(x+y)=\sin x\cos y+\cos x\sin y$
- $\sin(x-y)=\sin x\cos y-\cos x\sin y$
- $\cos(x+y)=\cos x\cos y-\sin x\sin y$
- $\cos(x-y)=\cos x\cos y+\sin x\sin y$
- $\tan(x+y)=\dfrac{\tan x+\tan y}{1-\tan x\tan y}$, $\tan(x-y)=\dfrac{\tan x-\tan y}{1+\tan x\tan y}$
- $\cot(x+y)=\dfrac{\cot x\cot y-1}{\cot y+\cot x}$, $\cot(x-y)=\dfrac{\cot x\cot y+1}{\cot y-\cot x}$

Example: If $\text{cosx}=-\frac{3}{5}$, x lies in the third quadrant, find the values of other five trigonometric functions.

Solution: Since $\text{cosx}=-\frac{3}{5}$, we have $\text{secx}=\frac{1}{\cos x}=-\frac{5}{3}$

Now

$$\sin^2\text{x}+\cos^2\text{x}=1\Rightarrow\sin^2\text{x}=1-\cos^2\text{x}$$

$$\sin^2\text{x}=1-\frac{9}{25}=\frac{16}{25}\Rightarrow\sin x=\pm\frac{4}{5}$$

Since x lies in the third quadrant, $\sin x$ in negative. Therefore $\sin x = -\frac{4}{5}$

Therefore $\operatorname{cosec}x = \frac{1}{\sin x} = -\frac{5}{4}$

Further, we have $\tan x = \frac{\sin x}{\cos x} = \frac{4}{3}$ and $\cot x = \frac{1}{\tan x} = \frac{3}{4}$

Example: Find the value of $\cos(-1710^0)$

Solution: We know that values of $\cos x$ repeats after an interval of 2π or 360^0.

Therefore, $\cos(-1710^0) = \cos(-1710^0 + 5\times 360^0) = \cos 90^0 = 0$

Example: Prove that $3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4} = 1$

Solution: We have, LHS = $3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4}$

$$= 3\times\frac{1}{2}\times 2 - 4\sin\left(\pi - \frac{\pi}{6}\right)\times 1 = 3 - 4\sin\frac{\pi}{6}$$

$$= 3 - 4\times\frac{1}{2} = 1 = \text{RHS}$$

Topics 4.3. Multiple and Submultiples Angles:

- $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1-\tan^2 x}{1+\tan^2 x}$
- $\sin 2x = 2\sin x\cos x = \frac{2\tan x}{1+\tan^2 x}$
- $\tan 2x = \frac{2\tan x}{1+\tan^2 x}$
- $\sin 3x = 3\sin x - 4\sin^3 x$
- $\cos 3x = 4\cos^3 x - 3\cos x$

Topics 4.4 Transformation from Product to Sum or Difference of Two Angles or Vice-Versa

- $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$
- $\sin x - \sin y = 2\sin\frac{x-y}{2}\sin\frac{x+y}{2}$

- $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$
- $\cos x - \cos y = 2\sin\frac{x+y}{2}\sin\frac{y-x}{2}$
- $2\sin x\cos y = \sin(x+y) + \sin(x-y)$
- $2\cos x\sin y = \sin(x+y) - \sin(x-y)$
- $2\cos x\cos y = \cos(x+y) + \cos(x-y)$
- $2\sin x\sin y = \cos(x-y) - \cos(x+y)$
- $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

Example: Show that $\tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$

Solution:

$$\tan 3x = \tan(2x + x) \Rightarrow \tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$
$$\tan 3x(1 - \tan 2x \tan x) = \tan 2x + \tan x$$
$$\tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$$
$$\tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$$

Example: Prove that $\frac{\cos 7x + \cos 5x}{\sin 7x - \sin 5x} = \cot x$

Answer:

$$\text{LHS} = \frac{\cos 7x + \cos 5x}{\sin 7x - \sin 5x} = \frac{2\cos\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)}{2\cos\left(\frac{7x+5x}{2}\right)\sin\left(\frac{7x-5x}{2}\right)}$$

$$= \frac{\cos x}{\sin x} = \cot x = \text{RHS}$$

Example: Prove that $\cos 2x\cos\frac{x}{2}-\cos 3x\cos\frac{9x}{2}=\sin 5x\sin\frac{5x}{2}$

Solution: We have

$$\text{LHS}=\cos 2x\cos\frac{x}{2}-\cos 3x\cos\frac{9x}{2}$$

$$=\frac{2\cos 2x\cos\frac{x}{2}}{2}-\frac{2\cos\frac{9x}{2}\cos 3x}{2}$$

$$=\frac{1}{2}\left[\cos\left(2x+\frac{x}{2}\right)+\cos\left(2x-\frac{x}{2}\right)-\cos\left(\frac{9x}{2}+3x\right)-\cos\left(\frac{9x}{2}-3x\right)\right]$$

$$=\frac{1}{2}\left[\cos\frac{5x}{2}+\cos\frac{3x}{2}-\cos\frac{15x}{2}-\cos\frac{3x}{2}\right]$$

$$=\frac{1}{2}\left[\cos\frac{5x}{2}-\cos\frac{15x}{2}\right]=\frac{1}{2}\left[-2\sin\left\{\frac{\frac{5x}{2}+\frac{15x}{2}}{2}\right\}\sin\left\{\frac{\frac{5x}{2}-\frac{15x}{2}}{2}\right\}\right]$$

$$=-\sin 5x\sin\left(-\frac{5x}{2}\right)=\sin 5x\sin\frac{5x}{2}=\text{RHS}$$

Example: Find the value of $\tan\frac{\pi}{8}$

Solution: Let $x=\frac{\pi}{8}$, Then $2x=\frac{\pi}{4}\Rightarrow\ \tan 2x=\frac{2\tan x}{1-\tan^2 x}\Rightarrow\tan\frac{\pi}{4}=\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$

$$\Rightarrow y=\tan\frac{\pi}{8},\text{then } 1=\frac{2y}{1-y^2}\Rightarrow y^2+2y-1=0$$

Therefore $\quad y=\frac{-2\pm 2\sqrt{2}}{2}=-1\pm\sqrt{2}$

Since $\frac{\pi}{8}$ lies in the first quadrant, $y=\tan\frac{\pi}{8}$ is positive

Hence, $\tan\frac{\pi}{8}=\sqrt{2}-1$